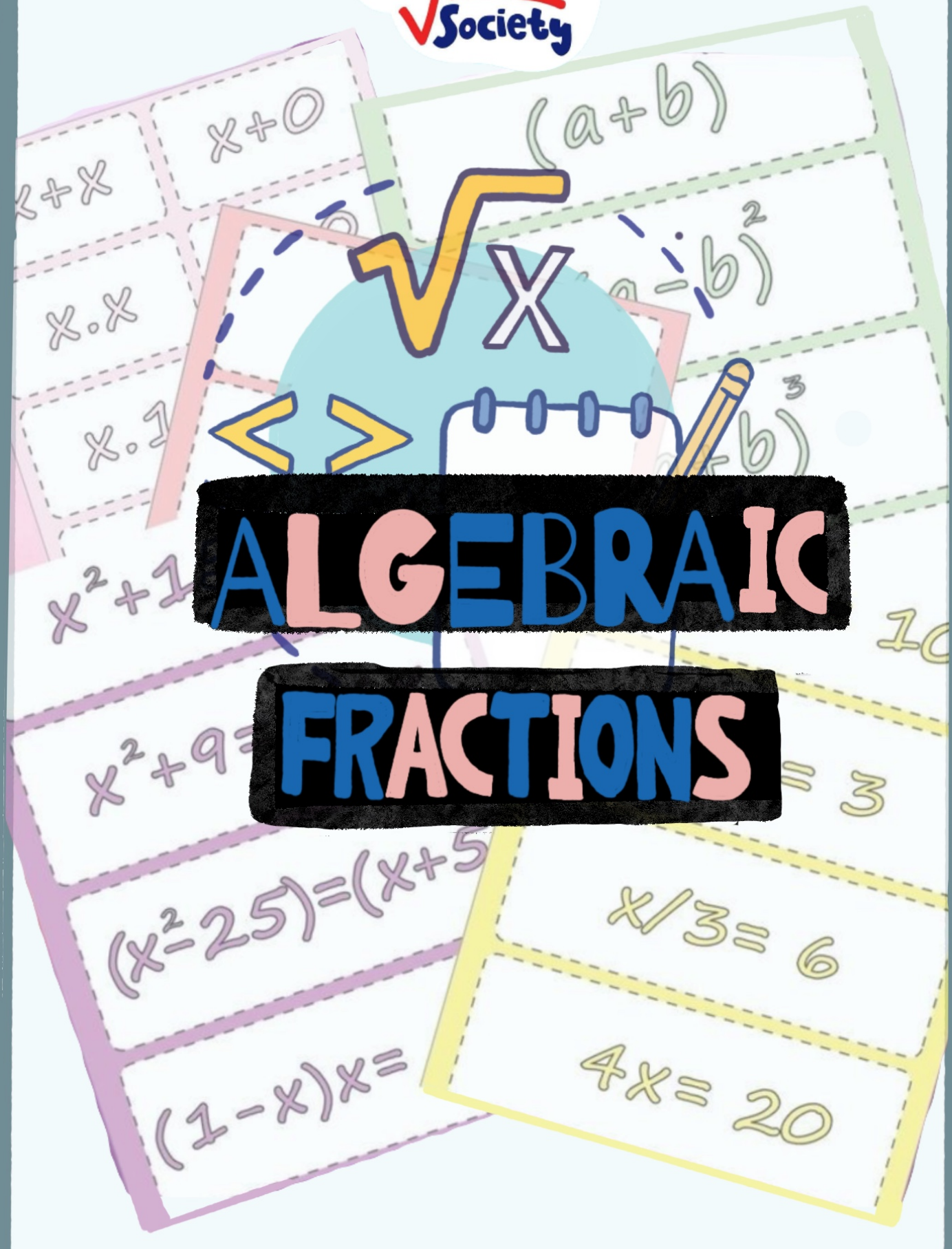
The Maths Society



ALGEBRAIC FRACTIONS

Question 1

Write as a single fraction in its simplest form.

$$\frac{2x-1}{3} - \frac{2}{x+1}$$ [3]

$\times (x+1)$ $\times 3$

$$= \frac{(2x-1)(x+1) - 2\times 3}{3(x+1)}$$

$$= \frac{2x^2+2x-x-1-6}{3(x+1)} = \frac{2x^2+x-7}{3(x+1)}$$

Question 2

Simplify. $\dfrac{x^2-16}{x^2-3x-4}$ [4]

$$\frac{(x-4)(x+4)}{(x-4)(x+1)} = \frac{x+4}{x+1}$$

Question 3

Write as a single fraction in its simplest form.

$$\frac{3}{x+2} - \frac{4}{2x-5}$$ [3]

$\times (2x-5)$ $\times (x+2)$

$$= \frac{6x-15-4x-8}{(x+2)(2x-5)} = \frac{2x-23}{(x+2)(2x-5)}$$

Question 4

(a) Write as a single fraction in its simplest form.

$$\frac{3}{2x-1} - \frac{1}{x+2}$$ [3]

$\times (x+2)$ $\times (2x-1)$

$$= \frac{3x+6-2x+1}{(2x-1)(x+2)}$$

$$= \frac{x+7}{(2x-1)(x+2)}$$

(b) Simplify.

$$\frac{4x^2-16x}{2x^2+6x-56}$$ [4]

$$\frac{4x(x-4)}{2(x^2+3x-28)} = \frac{\overset{2}{\cancel{4}}x(x-4)}{\cancel{2}(x+7)(x-4)} = \frac{2x}{(x+7)}$$

Question 5

Write as a single fraction, in its simplest form. [4]

$$\frac{3}{2x} + \frac{2x}{3} + 3 + 2x$$

$$= \frac{9}{6x} + \frac{4x^2}{6x} + \frac{3 + 2x}{}$$

$$= \frac{4x^2 + 9 + 18x + 12x^2}{6x}$$

$$= \frac{16x^2 + 18x + 9}{6x}$$

Question 6

Write as a single fraction in its simplest form.

$$\frac{2}{x} - \frac{2}{x+1}$$ [3]

$$\frac{2x + 2 - 2x}{x(x+1)} = \frac{2}{x(x+1)}$$

Question 7

Solve the equation.

$$\frac{3}{2x} + \frac{1}{x+1} = 0$$ [3]

$$\frac{3x + 3 + 2x}{2x(x+1)} = 0$$

$$6x + 3 = 0$$

$$6x = -3$$

$$x = -\frac{1}{2}$$

Question 8

Simplify.

$$\frac{x^2 + 6x - 7}{3x + 21}$$ [4]

$$\frac{(x+7)(x-1)}{3(x+7)} = \frac{x-1}{3}$$

Question 9

(a) Factorise $x^2 + x - 30$. [2]

$(x+6)(x-5)$

(b) Simplify $\dfrac{(x-5)(x+4)}{x^2+x-30}$. [1]

$\dfrac{(x-5)(x+4)}{(x+6)(x-5)} = \dfrac{x+4}{x+6}$

Question 10

Write as a single fraction in its simplest form. [3]

$\dfrac{2}{x+3} + \dfrac{3}{x+2}$ $\times x+2$ $\times x+3$

$= \dfrac{2x+4+3x+9}{(x+3)(x+2)}$

$= \dfrac{6x+13}{(x+3)(x+2)}$

Question 1

Write as a single fraction in its simplest form. [4]

$\dfrac{x+3}{x-3} - \dfrac{x-1}{x+1}$ $\times x+1$ $\times x-3$

$\dfrac{x^2+x+3x+3-x^2+3x+x-3}{(x-3)(x+1)}$

$\dfrac{8x}{(x-3)(x+1)}$

Question 2

Write the following as a single fraction in its simplest form. [3]

$\dfrac{x+2}{3} - \dfrac{2x-1}{4} + 1$ $\times 4$ $\times 3$ $\times 12$

$\dfrac{4x+8-6x+3+12}{12} = \dfrac{-2x+23}{12}$

Question 3

Simplify the following. $\dfrac{h^2-h-20}{h^2-25}$ [4]

$$\frac{(h-5)(h+4)}{(h-5)(h+5)} = \frac{h+4}{h+5}$$

Question 4

Simplify fully.

$$\frac{x^2-x-20}{x^3-10x^2+25x}$$ [5]

$$= \frac{(x-5)(x+4)}{x(x^2-10x+25)}$$

$$= \frac{(x-5)(x+4)}{x(x-5)(x-5)} = \frac{x+4}{x(x-5)}$$

Question 5

Write as a single fraction in its simplest form.

$$\frac{3}{x+10} - \frac{1}{x+4}$$ [3]

$\times x+4$ $\quad$ $\times x+10$

$$= \frac{3x+12-x-10}{(x+10)(x+4)} = \frac{2x+2}{(x+10)(x+4)}$$

$$= \frac{2(x+1)}{(x+10)(x+4)}$$

Question 6

Write the following as a single fraction in its simplest form.

$$\frac{x+1}{x+5} - \frac{x}{x+1}$$ [4]

$\times x+1$ $\quad$ $\times x+5$

$$= \frac{x^2+2x+1-x^2-5x}{(x+5)(x+1)}$$

$$= \frac{-3x+1}{(x+5)(x+1)}$$

Question 7

Write $\frac{2}{x-2}+\frac{3}{x+2}$ as a single fraction.

Give your answer in its simplest form. [3]

$\frac{2x+4+3x-6}{(x-2)(x+2)} = \frac{5x-2}{(x-2)(x+2)}$

Question 8

Write as a single fraction in its simplest form.

$\frac{2}{x}+\frac{1}{2x}+\frac{1}{2}$ [2]

$\frac{4+1+x}{2x} = \frac{5+x}{2x}$

Question 9

Simplify this fraction. $\frac{x^2-5x+6}{x^2-4}$ [4]

$\frac{(x-3)(x-2)}{(x-2)(x+2)}$

$= \frac{(x-3)}{(x+2)}$

Question 10

Write as a single fraction, in its simplest form.

$\frac{3}{x+2}-\frac{2}{x-1}$ [3]

$= \frac{3x-3-2x-4}{(x+2)(x-1)} = \frac{x-7}{(x+2)(x-1)}$

Question 1

(a) Write $\frac{1}{y} - \frac{2}{x}$ as a single fraction in its lowest terms. [2]

$\times x \quad \times y$

$$\frac{x - 2y}{xy}$$

(b) Write $\frac{x^2 + x}{3x + 3}$ in its lowest terms. [3]

$$\frac{x(x+1)}{3(x+1)} = \frac{x}{3}$$

Question 2

Write as a single fraction in its simplest form [2]

$$\frac{x}{3} + \frac{x-1}{2}.$$

$\times 2 \quad \times 3$

$$\frac{2x + 3x - 3}{6} = \frac{5x - 3}{6}$$

Question 3

Write as a single fraction in its simplest form

$$\frac{4}{2x+3} - \frac{2}{x-3}$$

$\times (2x-3) \quad \times (2x+3)$

[3]

$$\frac{4x - 12 - 4x - 6}{(2x+3)(x-3)}$$

$$= \frac{-18}{2x^2 - 3x + 9}$$

[2]

Question 4

Simplify $\frac{x}{3} + \frac{5x}{9} - \frac{5x}{18}$.

$\times 6 \quad \times 2 \quad \times 1$

$$\frac{6x + 10x - 5x}{18} = \frac{11x}{18}$$

Question 5

Write as a fraction in its simplest form

$\frac{x-3}{4} + \frac{4}{x-3}$. [3]

$\times (x-3) \quad \times 4$

$$\frac{x^2-6x+9+16}{4x-12}$$

$$= \frac{x^2-6x+25}{4x-12}$$

Question 6

Write as a single fraction in its simplest form

$\frac{5}{x} - \frac{4}{x+1}$. [2]

$\times x(x+1) \quad \times x$

$$\frac{5x+5-4x}{x(x+1)} = \frac{x+5}{x^2+x}$$

Question 7

Simplify

$\frac{\frac{x+2}{x} - \frac{x}{x+2}}{}$.

$\times x(x+2) \quad \times x$

Write your answer as a fraction in its simplest form. [3]

$$\frac{x^2+4x+4-x^2}{x(x+2)} = \frac{4x+4}{x^2+2x}$$

Question 8

(a) Write $\frac{3}{x} - \frac{2}{x+1}$ as a single fraction in its simplest form. [3]

$\times x(x+1) \quad \times x$

$$\frac{3x+3-2x}{x(x+1)} = \frac{x+3}{x^2+x}$$

(b) Solve the equation $\frac{3}{x} - \frac{2}{x+1} = 0$. [1]

$\times x(x+1) \quad \times x$

$$\frac{3x+3-2x}{x(x+1)} = 0$$

$$x+3 = 0$$

$$x = -3$$

Question 9

Work out as a single fraction $\frac{2}{x-3} - \frac{1}{x+4}$. [3]

$\times(x+4)$ $\times(x-3)$

$$\frac{2x+8-x+3}{(x+3)(x+4)} = \frac{x+11}{x^2+7x+12}$$

Question 10

Write $2x - \frac{10x}{5-x}$ as a single fraction. [2]

$\times(5-x)$ $\div 1$

$$\frac{10x-2x^2-10x}{5-x} = \frac{2x^2}{x-5} \text{ or } \frac{-2x^2}{5-x}$$

Question 1

Write as a single fraction in its simplest form. [4]

$\frac{5}{x-3} + \frac{3}{x+7} + \frac{1}{2}$

$\times 2(x+7)$ $\times 2(x-3)$ $\times(x-3)(x+7)$

$$\frac{10+70+6x-18+x^2+4x-21}{(x-3)(x+7)(2)}$$

$$= \frac{x^2+10x+41}{2x^2+8x-42}$$

Question 2

Write as a single fraction in its simplest form. [3]

$\frac{x+1}{x} - \frac{y-1}{y}$

$\times y$ $\times x$

$$\frac{xy+y-xy+x}{xy}$$

$$= \frac{x+y}{xy}$$

Question 3

Write as a single fraction in its simplest form. [3]

(a) $\frac{x^2 - 3x}{x^2 - 9}$

$$\frac{x(x-3)}{(x-3)(x+3)} = \frac{x}{x+3}$$

$\times 2x+5 \quad \times x-4$

(b) $\frac{3}{x-4} + \frac{2}{2x+5}$ [3]

$$\frac{6x+15+2x-8}{(x-4)(2x+5)} = \frac{8x+7}{2x^2-3x-20}$$

Question 4

Simplify. [2]

$\frac{x^3y+2xy^3}{x^2y^2}$

$$\frac{xy(x^2+2y^2)}{x\,x\,y\,y} = \frac{x^2+2y^2}{xy}$$

Question 5

Write as a single fraction. $\times pt \quad \times t \quad \times p$ [2]

$\frac{1}{1} - \frac{2}{p} - \frac{3}{t}$

$$\frac{pt-2t-3p}{pt}$$

Question 6

Simplify. $\frac{42np-7n}{12pt-2t+18mp-3m}$ [4]

$$\frac{7n(6p-1)}{2t(6p-1)+3m(6p-1)} = \frac{7n(6p-1)}{(2t+3m)(6p-1)} = \frac{7n}{2t+3m}$$

Question 7

Simplify. [4]

$$\frac{4+10w}{8-50w^2}$$

$$\frac{2(2+5w)}{2(4-25w^2)} = \frac{2+5w}{(2-5w)(2+5w)} = \frac{1}{2-5w}$$

Question 8

Write as a single fraction in its simplest form. [3]

$$3-\frac{t+2}{t-1}$$

$$\frac{3t-3-t-2}{t-1}$$

$$= \frac{2t-5}{t-1}$$

Question 9

Write as a single fraction, in its simplest form.

$$\frac{1-x}{x} - \frac{2+x}{1-2x}$$ [4]

$$\frac{1-3x+2x^2-2x-x^2}{x(1-2x)}$$

$$= \frac{x^2-5x-1}{x-2x^2}$$